

# Ilustrasi Pembuatan Kurva Bezier

[en.wikipedia.org/wiki/Bézier\\_curve](https://en.wikipedia.org/wiki/B%C3%A9zier_curve)

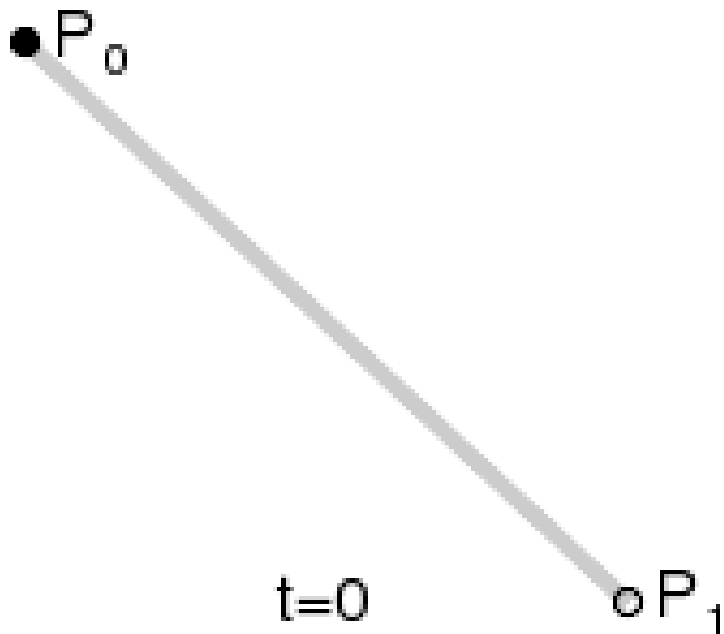
# Linear Bézier curves

- Given points  $\mathbf{P}_0$  and  $\mathbf{P}_1$ , a linear Bézier curve is simply a straight line between those two points. The curve is given by

$$\mathbf{B}(t) = \mathbf{P}_0 + t(\mathbf{P}_1 - \mathbf{P}_0) = (1 - t)\mathbf{P}_0 + t\mathbf{P}_1, t \in [0, 1]$$

- and is equivalent to linear interpolation.

# Ilustrasi: kurva linier



- The  $t$  in the function for a linear Bézier curve can be thought of as describing how far  $\mathbf{B}(t)$  is from  $\mathbf{P}_0$  to  $\mathbf{P}_1$ .
- For example when  $t=0.25$ ,  $\mathbf{B}(t)$  is one quarter of the way from point  $\mathbf{P}_0$  to  $\mathbf{P}_1$ .
- As  $t$  varies from 0 to 1,  $\mathbf{B}(t)$  describes a curved line from  $\mathbf{P}_0$  to  $\mathbf{P}_1$ .

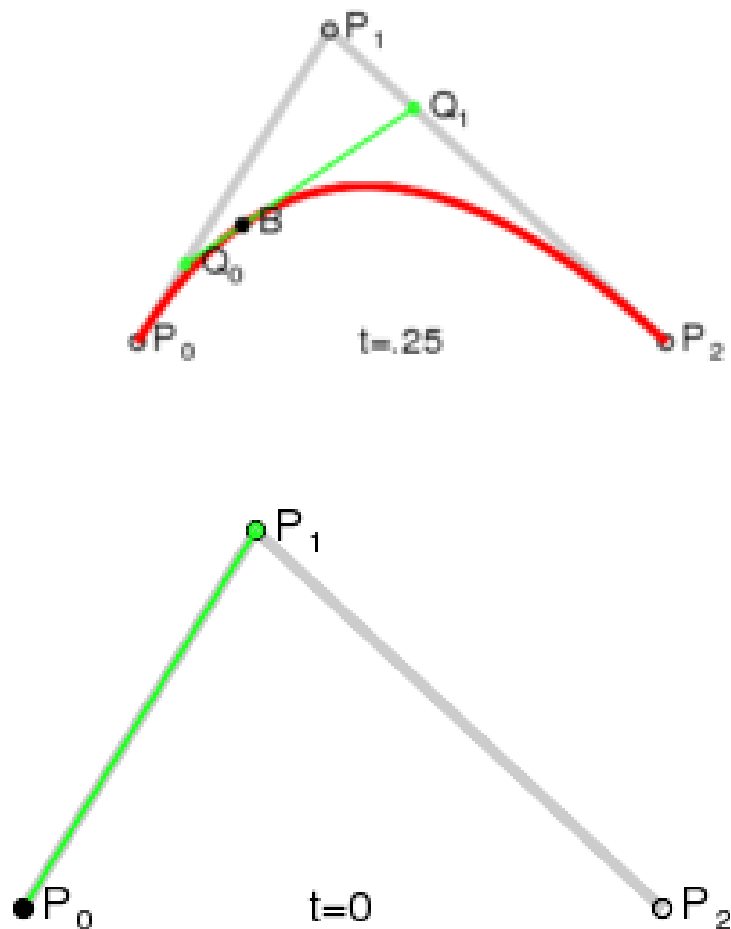
# Quadratic Bézier curves

- A quadratic Bézier curve is the path traced by the function  $\mathbf{B}(t)$ , given points  $\mathbf{P}_0$ ,  $\mathbf{P}_1$ , and  $\mathbf{P}_2$ ,

$$\mathbf{B}(t) = (1 - t)^2\mathbf{P}_0 + 2(1 - t)t\mathbf{P}_1 + t^2\mathbf{P}_2, \quad t \in [0, 1].$$

- A quadratic Bézier curve is also a parabolic segment.

# Ilustrasi: kurva kuadratik



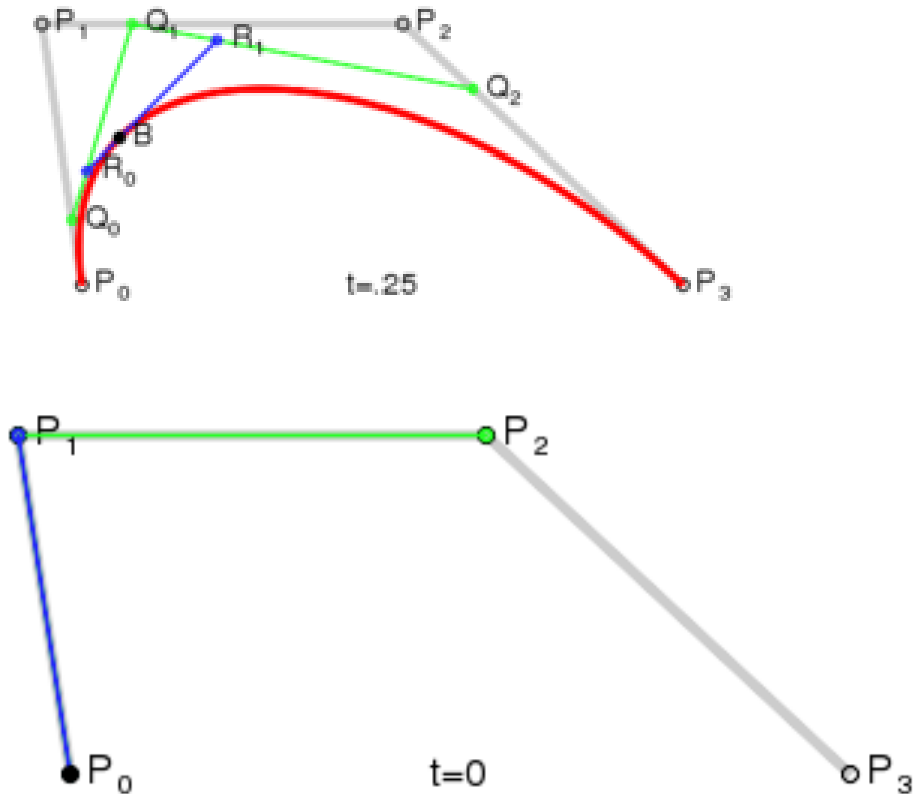
- For quadratic Bézier curves one can construct intermediate points  $Q_0$  and  $Q_1$  such that as  $t$  varies from 0 to 1:
- Point  $Q_0$  varies from  $P_0$  to  $P_1$  and describes a linear Bézier curve.
- Point  $Q_1$  varies from  $P_1$  to  $P_2$  and describes a linear Bézier curve.
- Point  $B(t)$  varies from  $Q_0$  to  $Q_1$  and describes a quadratic Bézier curve.

# Cubic Bézier curves

- Four points  $\mathbf{P}_0$ ,  $\mathbf{P}_1$ ,  $\mathbf{P}_2$  and  $\mathbf{P}_3$  in the plane or in three-dimensional space define a cubic Bézier curve.
- The curve starts at  $\mathbf{P}_0$  going toward  $\mathbf{P}_1$  and arrives at  $\mathbf{P}_3$  coming from the direction of  $\mathbf{P}_2$ . Usually, it will not pass through  $\mathbf{P}_1$  or  $\mathbf{P}_2$ ; these points are only there to provide directional information. The distance between  $\mathbf{P}_0$  and  $\mathbf{P}_1$  determines "how long" the curve moves into direction  $\mathbf{P}_2$  before turning towards  $\mathbf{P}_3$ .
- The parametric form of the curve is:

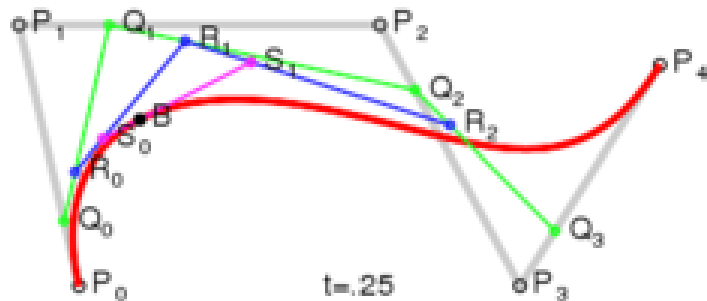
$$\mathbf{B}(t) = (1 - t)^3\mathbf{P}_0 + 3(1 - t)^2t\mathbf{P}_1 + 3(1 - t)t^2\mathbf{P}_2 + t^3\mathbf{P}_3, \quad t \in [0, 1].$$

# Ilustrasi : Kurva Cubic

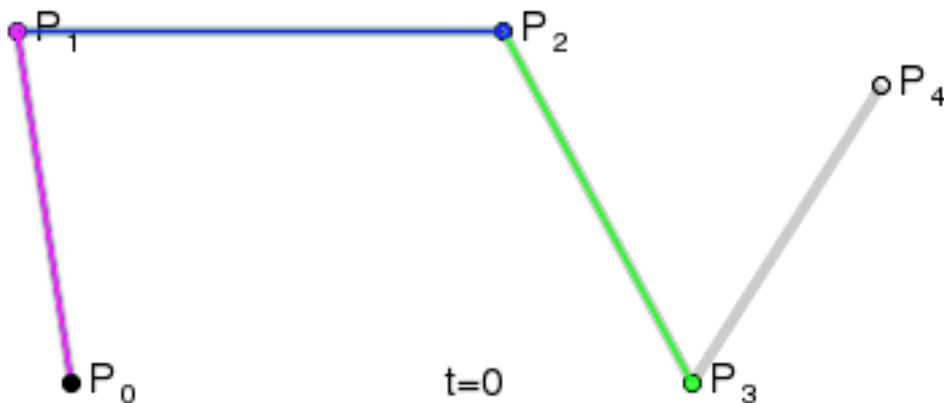


- For higher-order curves one needs correspondingly more intermediate points. For cubic curves one can construct intermediate points  $Q_0, Q_1$  &  $Q_2$  that describe linear Bézier curves, and points  $R_0$  &  $R_1$  that describe quadratic Bézier curves

# Ilustrasi: Fourth-order curve



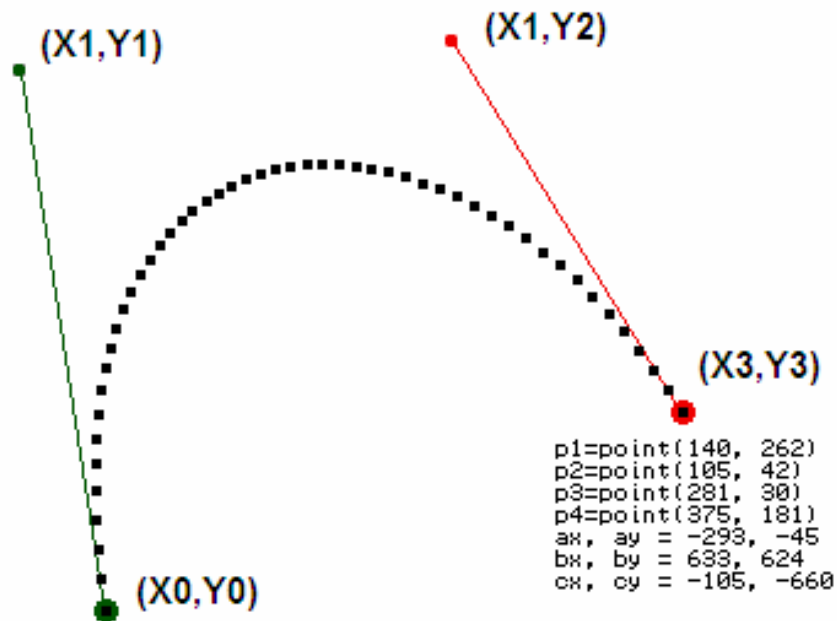
- For fourth-order curves one can construct intermediate points  $Q_0, Q_1, Q_2$  &  $Q_3$  that describe linear Bézier curves, points  $R_0, R_1$  &  $R_2$  that describe quadratic Bézier curves, and points  $S_0$  &  $S_1$  that describe cubic Bézier curves





# The Math Behind the Bezier Curve

[www.moshplant.com/direct-or/bezier/math.html](http://www.moshplant.com/direct-or/bezier/math.html)



- A cubic Bezier curve is defined by four points.
- Two are *endpoints*.
  - $(x_0, y_0)$  is the *origin* endpoint.
  - $(x_3, y_3)$  is the *destination* endpoint.
- The points  $(x_1, y_1)$  and  $(x_2, y_2)$  are *control points*.

- Two equations define the points on the curve. Both are evaluated for an arbitrary number of values of  $t$  between 0 and 1.
- As increasing values for  $t$  are supplied to the equations, the point defined by  $x(t), y(t)$  moves from the origin to the destination.

- This is how the equations are defined in Adobe's PostScript references.

$$x(t) = a_x t^3 + b_x t^2 + c_x t + x_0$$

$$x_1 = x_0 + c_x / 3$$

$$x_2 = x_1 + (c_x + b_x) / 3$$

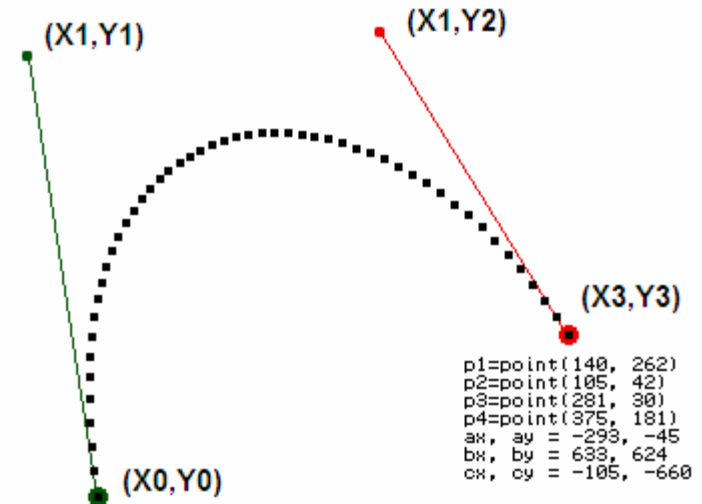
$$x_3 = x_0 + c_x + b_x + a_x$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + y_0$$

$$y_1 = y_0 + c_y / 3$$

$$y_2 = y_1 + (c_y + b_y) / 3$$

$$y_3 = y_0 + c_y + b_y + a_y$$



- This method of definition can be reverse-engineered, so:

$$c_x = 3 (x_1 - x_0)$$

$$b_x = 3 (x_2 - x_1) - c_x$$

$$a_x = x_3 - x_0 - c_x - b_x$$

$$c_y = 3 (y_1 - y_0)$$

$$b_y = 3 (y_2 - y_1) - c_y$$

$$a_y = y_3 - y_0 - c_y - b_y$$